Assessing Students' Conceptual Understanding of Calculus Using Item Response Theory

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Introduction

The Calculus Concept Inventory (CCI) is "a test of conceptual understanding of the most basic principles of differential calculus" (Epstein, 2013, p. 1018). Instructors have reported concerns that the CCI takes a large amount of instructional time to finish pre- and post-inventory.

To address this practical need, we developed a short version of the CCI (S-CCI) by selecting ten items from the original scale based on three criteria: (1) the items were as representative as possible, (2) the items were aligned with the canonical assessments for the course as well as the curriculum itself, and (3) having all students complete the assessment in under 30 minutes.

The study aims to:
(1) evaluate the item characteristics of the ten items in the S-CCI;
(2) compare normalized gain and IRT-estimated abilities as measures of learning gain;
(3) examine the effectiveness of different instructional approaches on conceptual understanding.

$$\text{Gain score} = \frac{\text{Posttest score} - \text{Pretest score}}{\text{Maximum possible score} - \text{Pretest score}}$$ VS $$P\left(X_j = 1 \middle| \theta, a_j, b_j, c_j\right) = c_j + (1 - c_j)\frac{e^{a_j(\theta - b_j)}}{1 + e^{a_j(\theta - b_j)}}.$$

IRT: three-parameter logistic model

Lecture-based (LB) sections VS Interactively-engaged (IE) sections

Method

Participants

Participants were 1,188 students (578 females, 610 males) enrolled in an Applied Calculus I course, which is taught for business or social science majors. A total of 1,113 students enrolled in LB sections while 75 students enrolled in IE sections.

LB sections	IE sections
Meet 3 times a week for 50 minutes each	Have one 50-minute IE class a week; students were highly encouraged to discuss and collaborate with their classmates as they work on the problems.

Procedure

All students were required to take the S-CCI twice (at the beginning and the end of the semester) as part of the course. These students also completed a survey about their perceptions of knowledge transfer (8-item scale). The course grade was obtained from the Office of the Registrar. Students received a minimal amount of extra credit points for participating in the study.

Results

Table 1

The Discrimination (a_j), Difficulty (b_j), and Guessing Chances (c_j) of the Posttest S-CCI

Item	a_j		b_j		c_j	
	Estimate	*SE*	Estimate	*SE*	Estimate	*SE*
1	2.59	1.21	0.88	0.09	0.07	0.06
2	2.03	0.48	0.45	0.17	0.10	0.08
3	0.31	0.08	-0.12	0.19	0.00	0.00
4	1.45	0.50	0.35	0.41	0.40	0.12
5	2.04	0.59	1.18	0.09	0.08	0.04
6	1.36	0.46	1.17	0.17	0.19	0.07
7	1.29	0.59	1.88	0.22	0.22	0.05
8	1.47	1.01	1.39	0.21	0.29	0.10
9	1.67	0.78	1.17	0.24	0.40	0.07
10	1.88	0.55	1.18	0.10	0.13	0.04

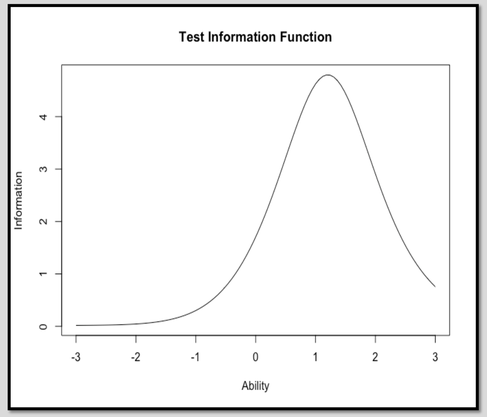

Figure 1. The test information function for the posttest data.

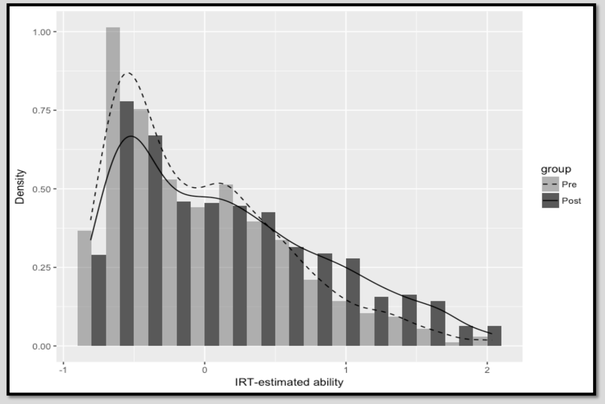

Figure 2. The distributions of pre- and post-instruction abilities for all 1,188 students.

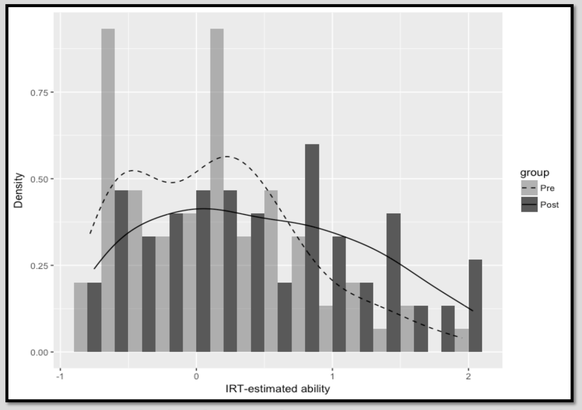

Figure 3. The distributions of pre- and post-instruction abilities for 75 students who were in the IE sections.

Conclusion

- S-CCI is sufficiently capable of assessing students' conceptual understanding of calculus, especially for students with high-level abilities.
- The IRT-estimated gain is a stronger predictor of academic outcomes than the normalized gain.
- Students in the interactively-engaged sections achieved greater gains than the students in the lecture-based sections.